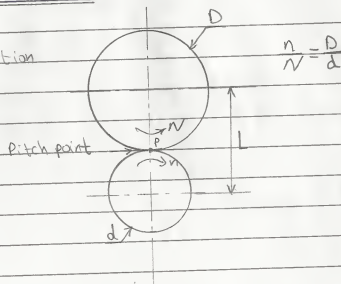


## 7. Gear measurement

### \* Friction wheels

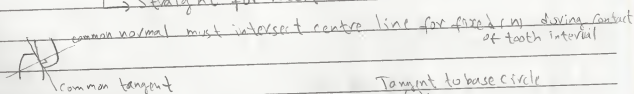
For low power & high friction transmission



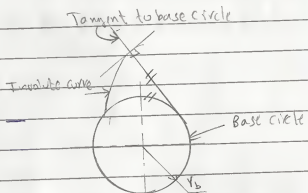
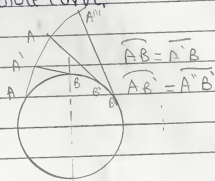
### \* Gears:

Positive transmission in which high power is transmitted

Tooth curves → Involute → Easier to manufacture and wear ☐  
 → Cycloidal → Difficult to manufacture of strong (casting) ☐  
 → Straight for rack



### \* The involute curve:



For any point on involute

Circle a wire around a disk. Pull the wire and remove it as well so wire end draws involute

Move a pencil round a disk to draw the involute curve

Tangent on base circle is involute curve

Normal to involute curve is tangent to base circle

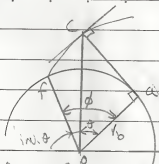
## \* Involute function:

From definition:  $\widehat{ac} = \widehat{af}$ 

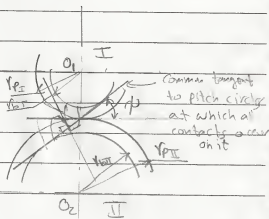
$$\text{inv. } \theta = \phi - \theta = \frac{\widehat{fa}}{r_b} - \theta = \frac{\widehat{ac}}{r_b} - \theta$$

$$\text{inv. } \theta = \tan \theta - \theta$$

$$\text{For } \psi = 20^\circ \text{ (pressure angle)} : \text{inv. } \theta = \tan 20^\circ - \frac{\pi \times 20^\circ}{180^\circ} = 0.0149$$

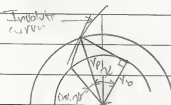


## \* The pressure angle

 $\psi \Rightarrow$  Thrust  $F \downarrow$  but it's weak


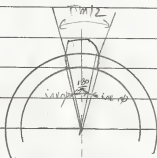
## \* Relation between pressure angle of circles:

$$\cos \psi = \frac{r_b}{r_p}$$



## \* Angle of tooth on base circle:

$$\theta = \frac{180}{\pi} + 12 \text{ inv. } \psi$$

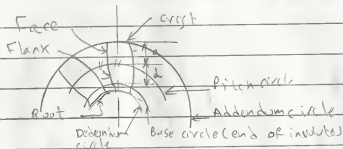


### \* Parameters of gear

$$a = m$$

$$d = 1.25m \text{ (for clearance)}$$

$$\text{Tooth depth} = a + d = 2.25m$$



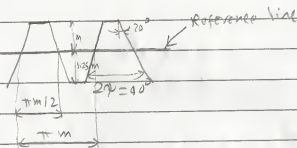
### \* Experimental prediction of gear module

1. Measure addendum circle ( $D_a$ )
2. Count the no. of teeth ( $Z$ )
3. Determine the gear module:  $D_a = mZ + 2m = m(Z + 2)$
4. Select the nearest standard module

### \* Involute rack:

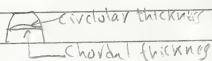
Tooth thickness (arc) at pitch circle

$$\frac{\pi D_p}{2Z} = \frac{\pi m Z}{2Z} = \frac{\pi m}{2}$$

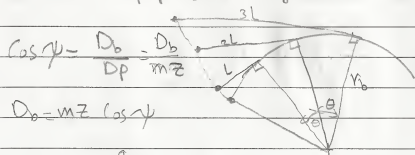


### \* Experiment: Check the form of the tooth flank (involute)

1. Determine the module of gear
2. Draw the profile of flank of tested gear by
  - A. Projecting the profile of gear tooth flank using projector
  - B. Measuring the tooth thickness (Chordal) at different tooth depths using gear tooth vernier, T.M.M, etc.



3. Draw the theoretical involute curve for the given module and pressure angle



$$\cos \phi = \frac{D_b}{D_p} = \frac{D_b}{mZ}$$

$$D_b = mZ \cos \phi$$

$$L = r_b \times \frac{\theta \pi}{180^\circ}$$

$$\theta \rightarrow 5^\circ$$

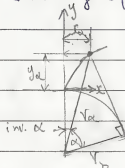
4. Compare the actual profile with the theoretical one

\* Drawing of involute curve without drawing of base circle:

$$x_a = r_a \sin(\text{inv. } \alpha)$$

$$y_a = r_a \cos(\text{inv. } \alpha) - r_b$$

$$r_a = \frac{r_b}{\cos \alpha} \Rightarrow \alpha = \cos^{-1} \left( \frac{r_b}{r_a} \right)$$



$$\text{in } r_a, \alpha = \tan^{-1} \alpha \text{ rad.}$$

$$\cos^{-1} \frac{r_b}{r_a} = \tan^{-1} \alpha \text{ rad.}$$

$r_i$	$\alpha$	$\text{inv. } \alpha$	$x_a$	$y_a$
$r_d$				
$r_p$				
$r_a$				

$$r_d = \frac{mZ}{2} - 1.25m = \frac{m}{2} (Z - 2.5)$$

$$r_p = \frac{mZ}{2}$$

$$r_a = \frac{mZ}{2} + m = \frac{m}{2} (Z + 2)$$

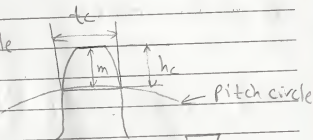
\* Chordal thickness:

(tc) Chordal thickness of pitch circle

(hc) Chordal height

$$t_c = 2 r_p \sin\left(\frac{90^\circ}{z}\right) = 2\left(\frac{mz}{2}\right) \sin\left(\frac{90^\circ}{z}\right)$$

$$t_c = mz \sin\left(\frac{90^\circ}{z}\right)$$



$$h_c = m + \left[ r_p - r_p \cos\left(\frac{90^\circ}{z}\right) \right]$$

$$h_c = m + \frac{mz}{2} \left[ 1 - \cos\left(\frac{90^\circ}{z}\right) \right]$$

Experimental procedure:

1. Measure outer diameter
2. Count no. of teeth
3. Determine the module
4. Approximate to the nearest standard module
5. Calculate the chordal height ( $h_c$ )
6. Set the vertical vernier of gear tooth vernier on ( $h_c$ ) value
7. Set it on the tooth tip as shown in figure & measure its thickness ( $t_m$ )



8. Calculate the chordal thickness theoretically:  $t_c = mz \sin\left(\frac{90^\circ}{z}\right)$
9. Determine the error in chordal thickness  $\Delta t_m = t_m - t_c$

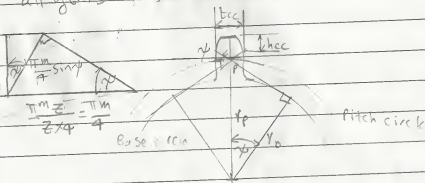
\* Constant Chord:

(hcc) is constant for all gears of same module

$$h_{cc} = m \frac{\pi m}{4} \cos \psi \sin \psi$$

$$h_{cc} = m \left( 1 - \frac{\pi}{4} (\cos \psi \sin \psi) \right)$$

$$t_{cc} = \frac{\pi m}{2} \cos^2 \psi$$

\* Experimental procedure:

1.  $D_o$
2.  $Z$
3.  $m$
4.  $m_{st}$
5.  $h_{cc}$ , calculated
6.  $t_{cc}$ , measured by gear tooth vernier
7.  $t_{cc}$ , calculated
8.  $\Delta = t_{ccm} - t_{cc}$

\* Base pitch: Normal distance between 2 invol.

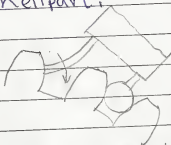
$$AB = a'b' = a'b'$$

$\hookrightarrow P_b$  (Base pitch)

$$\overline{a'c} = \overline{AC}, \overline{b'c} = \overline{BC}$$

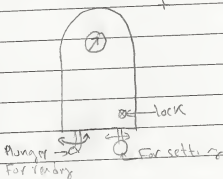
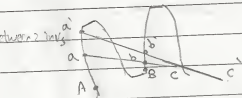
$$P_b = \frac{\pi D_b}{Z} = \frac{\pi m Z \cos \psi}{Z} = \pi m \cos \psi$$

Keilhart:



Move down or up for direction  
and read min. reading

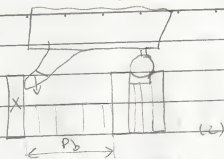
(Rg).





$$P_{bm} = \overset{D_o}{D_o} + P_b$$

$$P_{bm} = (\underset{\substack{\uparrow \\ \text{on gear}}}{R_g} - \underset{\substack{\uparrow \\ \text{Calculated}}}{R_{std}}) + \underset{\substack{\uparrow \\ \text{Initial}}}{A_b}$$



### Experimental procedure:

1.  $D_o$
2.  $Z$
3.  $m$
4.  $m_{std}$
5. Calculate base pitch:  $P_b = \pi m \cos \gamma$
6. Select the suitable nail part according to module ( $m$ )
7. Set the nail part opening to be suitable to measured gear
8. Take the reading on gear tooth as in (1) [least reading]
9. Build a block gauge combination as in (2)
10. Using nail part to take reading on combination ( $R_{std}$ ) [least reading]
11.  $\Delta = R_g - R_{std}$
12.  $P_{bm} = (R_g - R_{std}) + A_b$

Error  $\rightarrow$  Error in division during manufacturing  
 $\rightarrow$  Error in base circle

### \* Determination of error in pitch circle:

1. Take reading over all teeth of gear
2. Consider average of readings
3. Compare with standard reading  $\rightarrow R_{av} - R_{std}$

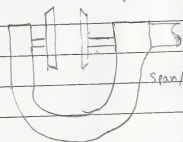
$$S_{av} = R_{av} - R_{std} \rightarrow D_o = \frac{(S_{av} + P_b)Z}{\pi}$$

$$S_{D_o} = D_{bm} - m Z \cos \gamma = Z S_{av}$$

Subject: \_\_\_\_\_

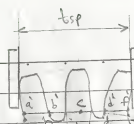
\* Span length

Anvil design ensures tangency



Span/gear mesh

Use anvil for large gear



Date: \_\_\_\_\_  
Tangency height up to

Base pitch

Base circle

$$\bar{a}c = \bar{a}c$$

$$\bar{a}f' = \bar{a}f$$

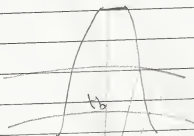
$$t_{sp} = \bar{a}f$$

$$t_{sp} = (n_{sp} - 1)P_b + t_b ; n_{sp} = \frac{Z}{g} \rightarrow \text{Approximate to larger}$$

$$P_b = t_{sp, n+1} - t_{sp, n}$$

$$\frac{360}{20 \times 2} = 9$$

$$t_{sp} = (n_{sp} - 1)\pi m \cos \psi + m \left( Z \cos \psi \sin \psi + \frac{\pi}{2} \cos \psi \right)$$



$$t_b = 2 \left( r_b \sin \psi + \frac{2\pi r_b}{4 \times Z} \right)$$

$$\psi = \sin^{-1} \frac{r_b}{mZ}$$

$$t_b = 2 \left( \frac{mZ \cos \psi \sin \psi}{2} + \frac{2\pi mZ \cos \psi}{4 \times Z} \right)$$

$$t_b = mZ \sin \psi \cos \psi + \frac{\pi m \cos \psi}{2}$$